Vertical Product Differentiation

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Reference

A. Shaked - J. Sutton, Relaxing Price Competition through Product Differentiation, Review of Economic Studies, 1982

Structure of the Game

Consider a three-stage game where firms :
- choose whether to enter the industry or not
- choose qualities of their product
- choose prices

Price Competition Stage - 1

Take n firms with products of quality such that

$u_1 < u_2 < u_3 < \ldots u_n$

each sold at price P_k (k=1,…,n)

There exists a continuum of consumers with identical tastes but different incomes t.

Incomes are uniformly distributed with density S (where S stands for "size of the market") on a support:

$0 < a \leq t \leq b$

Consumers buy at most one unit of the good.

Price Competition Stage - 2

In case of no purchase, their utility is:

$U\ (t,0)=u_0 * t$

(note that in Hotelling consumers had to buy, here they may not).

If they buy, consumers have utility

$U\ (t,k)=u_k * (t-p_k)$

(where it is assumed $t > p_k$).

Define

$$c_k = \frac{u_k}{u_k - u_{k-1}}$$

i.e. $c_k > 1$ is the inverse of the relative gain in quality from u_k and u_{k-1}

Price Competition Stage - 3

so that

$$1-c_k = 1-\frac{u_k}{u_k-u_{k-1}} = \frac{u_k-u_{k-1}-u_k}{u_k-u_{k-1}} = \frac{-u_{k-1}}{u_k-u_{k-1}}$$

As usual in “spatial models”, to build demand functions one derives the indifferent consumer.

A consumer indifferent between buying from k or k-1 is denoted as t_k and satisfies :

$u_k * (t_k - p_k) = u_{k-1} * (t_k - p_{k-1})$

where

$$t_k = p_k \underbrace{\frac{u_k}{u_k-u_{k-1}}}_{c_k} - p_{k-1} \underbrace{\frac{u_{k-1}}{u_k-u_{k-1}}}_{-(1-c_k)}$$

Price Competition Stage - 4

so that

$$t_k = p_k c_k + p_{k-1}(1 - c_k) \qquad k=2,3,\ldots,n$$

The consumer indifferent between to buy and not to buy is denoted as t_1 and satisfies:

$u_1 * (t_1 - p_1) = u_0 * t_1$

from which

$$t_1 = \frac{p_1 u_1}{\underbrace{u_1 - u_0}_{c_1}}$$

so that

$$t_1 = p_1 c_1$$

(note that it is as above with $p_0 = 0$**)**

Price Competition Stage - 5

Note that the values t_k just found partition all the consumers into segments corresponding to successive market shares of the firms.

This is easy to see when considering who prefers a good k to a good k-1:

$u_k * (t-p_k) > u_{k-1} * (t-p_{k-1})$

$\rightarrow$

$$t > p_k c_k + p_{k-1}(1-c_k) = t_k$$

All the people with income $t > t_k$ will prefer u_k to u_{k-1} and so on.

Price Competition Stage - 6

This way we can derive the firms demand functions:

$$q_1 = \begin{cases} (t_2 - a)S \text{ if } t_1 \leq a \\ (t_2 - t_1)S \text{ if } t_1 \geq a \end{cases}$$

$$q_k = (t_{k+1} - t_k)S \text{ with } 1 < k < n$$

$$q_n = (b - t_n)S$$

Assume now that the variable costs of production do not depend on quality (this is a crucial assumption for the finiteness property derived below) and in particular that they are zero (this is without loss of generality).

Price Competition Stage - 7

The profits (and revenue) functions are given by:

$$\Pi_1 = R_1 = \begin{cases} p_1(t_2 - a)S \text{ if } t_1 \leq a \\ p_1\,(t_2 - t_1)S \text{ if } t_1 \geq a \end{cases}$$

$$\Pi_k = R_k = p_k(t_{k+1} - t_k)S \quad \text{with } 1 < k < n$$

$$\Pi_n = R_n = p_n(b - t_n)S$$

Claim:

1) **At equilibrium (if it exists), the top quality product enjoys a positive market share.**

2) **Further, if any product has zero market share, a lower quality will also have zero market share.**

Price Competition Stage - 7

Proof :

Suppose the first quality from the top not to be sold is u_j. This means that people with $t > t_j$ **prefer $u_{j+1},\ldots, u_{j+n}$.**

But it must also be that all people with $t < t_j$ will also buy $u_{j+1},\ldots, u_{j+n}$.

Indeed, if there was a firm selling at a positive price a positive quantity of a product of quality u_{j-1} , then firm j could charge the same price as firm j-1 and will have all the customers of firm j-1 , since it sells a superior quality. Hence, this would not be a price equilibrium.

Price Competition Stage - 8

Imagine that n firms coexist as an equilibrium (with a positive market share, since they have to cover a fixed cost of entry, F).

The first order conditions of the last stage of the game are derived as follows:

- **<u>for k=2,…,n-1</u>**

$R_k = p_k(t_{k+1} - t_k)S$

where

$$t_{k+1} = p_k(1 - c_{k+1}) + p_{k+1}c_{k+1}$$

$$t_k = p_{k-1}(1 - c_k) + p_k c_k$$

That is t_k and t_{k+1} are functions of p_k.

Price Competition Stage - 9

$$\frac{\partial R_k}{\partial p_k} = S\left(t_{k+1} - t_k + p_k \frac{\partial t_{k+1}}{\partial p_k} - p_k \frac{\partial t_k}{\partial p_k} \right) = 0$$

which implies:

$$t_{k+1} - t_k + p_k(1 - c_{k+1}) - p_k c_k = 0$$

- **for k=1**

If $t_1 \leq a$

$R_1 = p_1(t_2 - a)S$

where

$$t_2 = p_1(1 - c_2) + p_2 c_2$$

That is t_2 is a function of p_1.

Price Competition Stage - 10

$$\frac{\partial R_1}{\partial p_1} = S\left(t_2 - a + p_1 \frac{\partial t_2}{\partial p_1} \right) = 0$$

which implies:

$$t_2 - a + p_1(1 - c_2) = 0$$

If $t_1 \geq a$

$R_1 = p_1(t_2 - t_1)S$

where

$$t_2 = p_1(1 - c_2) + p_2 c_2$$

$$t_1 = p_1 c_1$$

That is t_2 and t_1 are functions of p_1.

Price Competition Stage - 11

$$\frac{\partial R_1}{\partial p_1} = S\left(t_2 - t_1 + p_1 \frac{\partial t_2}{\partial p_1} - p_1 \frac{\partial t_1}{\partial p_1} \right) = 0$$

which implies:

$$t_2 - t_1 + p_1(1 - c_2) - p_1 c_1 = 0$$

- **for k=n**

$R_n = p_n(b - t_n)S$

where

$$t_n = p_{n-1}(1 - c_n) + p_n c_n$$

That is t_n is a function of p_1.

Price Competition Stage - 12

$$\frac{\partial R_n}{\partial p_n} = S\left(b - t_n - p_n \frac{\partial t_n}{\partial p_n} \right) = 0$$

which implies:

$$b - t_n - p_n c_n = 0$$

Lemma: Let b<4a. Then for any Nash equilibrium involving the distinct goods n, n-1,…,1 at most two products have a positive market share at equilibrium.

Price Competition Stage - 13

Proof: (by contradiction)

Suppose there are three or more products at the Nash Equilibrium.

The f.o.c. are then given by:

$$t_{k+1} - t_k + p_k(1 - c_{k+1}) - p_k c_k = 0$$

Note from the definition of indifferent consumer that

$$t_k = p_k c_k + p_{k-1}(1 - c_k)$$

$$\rightarrow$$

$$p_k c_k = t_k - p_{k-1}(1 - c_k) p_{k-1}$$

Replacing the latter into the former you get

Price Competition Stage - 14

$$t_{k+1} - 2t_k + p_k \underbrace{(1 - c_{k+1})}_{<0} + p_{k-1} \underbrace{(1 - c_k)}_{<0} = 0 \qquad \textbf{(A)}$$

Using the same trick the f.o.c. for firm n is

$$b - 2t_n + p_{n-1} \underbrace{(1 - c_{n-1})}_{<0} = 0 \qquad \textbf{(B)}$$

So that

$$t_{k+1} - 2t_k > 0$$

and

$$b - 2t_n > 0$$

Price Competition Stage - 15

But

$$t_{k+1} - 2t_k > 0 \rightarrow t_n - 2t_{n-1} > 0 \rightarrow 2t_n - 4t_{n-1} > 0$$

Since $b - 2t_n > 0$**, then** $b > 2t_n > 4t_{n-1}$

However, by assumption, $b < 4a$

So that

$$4a > 4t_{n-1} \rightarrow a > t_{n-1}$$

But this means that the consumer indifferent between u_{n-1} and u_{n-2} is located before a, where there exists no consumer

Nobody will buy from the third quality firm (everybody buys either u_n or u_{n-1})

Price Competition Stage - 16

We have then found a contradiction: more than two products cannot coexist at the Nash equilibrium in prices.

Some comments:

-**The intuition behind this result is that price competition between "high quality" producers drives prices so low that nobody will wish to buy the lower quality goods, not even if they were sold at zero prices.**

-**Note that the number of products which can survive at equilibrium depends just on the distribution of income (for $b/4 < a < b/2$ exactly two firms exist in the industry, for instance).**

Price Competition Stage - 17

This result sets the basis for an important property of vertical product differentiation models, that is the <u>finiteness property</u>, which says that even as the size of the market tends to infinity, there exists an upper bound to the number of firms which can survive in the industry. (see Sutton, Sunk Cos and Market Structure, MIT Press , 1991).

(In our case the number of firms is even independent of S!)

Quality Competition Stage - 1

To analyze the quality competition stage, let's make some simplifying assumptions. We first go through the price competition stage under these simplifying assumptions.

Assume n=2 and b< 4a.

Assume the market is covered for simplicity.

Then, demand functions are:

$q_1 = (t_2 - a)S$ **and** $q_2 = (b - t_2)S$

Then

$\Pi_1 = p_1(t_2 - a)S$ **and** $\Pi_2 = p_2(b - t_2)S$

Substituting for t_1 and t_2:

$\Pi_1 = p_1[p_1(1-c_2) + p_2 c_2 - a]S$ **and** $\Pi_2 = p_2[b - p_1(1-c_2) - p_2 c_2]S$

Quality Competition Stage - 2

$$\begin{cases} \dfrac{\partial \Pi_1}{\partial p_1} = 0 & \rightarrow \; 2p_1(1-c_2)+p_2c_2-a=0 \\ \dfrac{\partial \Pi_2}{\partial p_2} = 0 & \rightarrow \; b-p_1(1-c_2)-p_2c_2=0 \end{cases}$$

$$\begin{cases} p_1^* = \dfrac{b-2a}{3(c_2-1)} \\ p_2^* = \dfrac{2b-a}{3c_2} \end{cases}$$

(you can check that $p_2^* > p_1^*$ **and** $q_2^* > q_1^*$ **; recall that** $u_2 > u_1$**)**

Quality Competition Stage - 3

$$\begin{cases} \Pi_1^* = \dfrac{(b-2a)^2}{9(c_2-1)} \\ \Pi_2^* = \dfrac{(2b-a)^2}{9c_2} \end{cases}$$

Recall that

$$c_2 = \frac{u_2}{u_2 - u_1}$$ **and** $$c_2 - 1 = \frac{u_1}{u_2 - u_1}$$

Let us now analyse the quality sub-game

Quality Competition Stage - 4

$$\begin{cases} \Pi_1^* = \dfrac{(b-2a)^2}{9} \dfrac{u_2 - u_1}{u_1} \\ \Pi_2^* = \dfrac{(2b-a)^2}{9} \dfrac{u_2 - u_1}{u_2} \end{cases}$$

$$\begin{cases} \dfrac{\partial \Pi_1^*}{\partial u_1} = \dfrac{-(b-2a)^2}{9} \dfrac{u_2}{u_1^{\ 2}} < 0 \\ \dfrac{\partial \Pi_2^*}{\partial u_2} = \dfrac{(2b-a)^2}{9} \dfrac{u_1}{u_2^2} > 0 \end{cases}$$

⇨ **Firms differentiate as much as they can (in fact in Shaked-Sutton results are slightly different: u_2 takes the maximum allowed value, but u_1 does not fall down to u_o to avoid competition with the no-buy “fall-back” quality)**

Quality Competition Stage - 5

A similar result (firms differentiate their products to relax price competition) is obtained in a slightly different type of vertical product differentiation models, à la Mussa-Rosen (1978, Monopoly and Product Quality, Journal of Economic Theory), where:

U=θs-p

with

- $\theta \in (\theta_{min}, \theta_{max})$ **taste for quality**

-**s quality**

-**p price**

See also, J.Tirole, The Theory of Industrial Organization, The Mit Press, 1988, pages 296-298 and Belleflamme and Peitz, , Industrial Organization: Markets and Structure, Cambridge University Press Cap.5.3.1